Pressure and Wojnow's Thermodynamic Cosmic Entropy in the $R_H = ct$ Framework

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Abstract

This document addresses the question: *"Can we associate a pressure with Wojnow's entropy in the $R_H = ct$ reference framework?"* Within the model-specific assumptions adopted here, we define a trajectory effective pressure by $P_{eff} = -dE_{R_H}/dV_H$. We show that this quantity can be expressed in terms of the product $T_{R_H}S_{R_H} = E_{R_H}$. The resulting formula is an algebraic representation along the $R_H = ct$ cosmological trajectory and is not presented as an independent derivation of an equilibrium thermodynamic pressure.

Keywords: Cosmology, $R_H = ct$ model, Thermodynamic Cosmic Entropy, Wojnow Entropy, Trajectory Effective Pressure, Critical Energy Density, Hubble Radius, CMB Temperature

1. Introduction: The Original Question

The starting point of this analysis is the following question, posed in the context of the $R_H = ct$ cosmological framework:

"Normally, one should be able to associate a pressure with Wojnow's entropy in this reference framework. How?"

This question arises from the thermodynamic treatment of the universe in the $R_H = ct$ model, where the cosmic microwave background (CMB) temperature T_{CMB} and the critical energy density $\rho_{E,c}$ are fundamental quantities. The thermodynamic cosmic entropy S_{R_H}, introduced by Wojnow (see [10] and also for explicit relationship between H and T_{CMB}), plays a pivotal role in linking these quantities through the model-specific thermodynamic relations adopted here together with standard gravitational relations.

The $R_H = ct$ model, first proposed by Fulvio Melia [1,2,3], offers a compelling alternative to the standard ΛCDM paradigm. In this framework, the equality $R_H = ct$ (where R_H is the Hubble radius and t is cosmic time) is not a coincidence of the present epoch but a fundamental relativistic constraint derived from Birkhoff's theorem and the Weyl postulate. In the field of this model, Melia has demonstrated consistency with a wide range of cosmological observations, including supernovae data and baryon acoustic oscillations.

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The present document provides a step-by-step derivation of the pressure associated with Wojnow's entropy.

2. Theoretical Framework and Foundational Formulas

2.1 The $R_H = ct$ Cosmological Model

In the $R_H = ct$ framework, the Hubble radius is defined as:

$$R_H = c/H = c\,t$$

where H is the Hubble parameter and t is cosmic time. This equality holds for all cosmic time in a flat universe, representing a fundamental constraint rather than a temporal coincidence.

2.2 Energy at the Hubble Horizon

By using the Schwarzschild-type mass–radius relation as a model-specific Hubble-horizon identification, the enclosed energy is written as:

$$E_{R_H} = \frac{c^4 R_H}{2G} \tag{2.1}$$

This formula arises from a growing black hole universe analogy:

- For a Schwarzschild black hole: $E = M\,c^2$, with $M = \frac{c^2 R_S}{2G}$ (Schwarzschild mass) with $R_S = R_H = ct$.
- Thus, $E_{R_H} = M\,c^2 = \frac{c^4 R_H}{2G}$.

2.3 Thermodynamic Relationships

2.3.a Model-Specific Thermodynamic Identification

We adopt $E_{R_H} = T_{R_H} S_{R_H} = c^4\,R_H/2\,G$ as a model-specific thermodynamic identification :

$$E_{R_H} = T_{R_H} S_{R_H} \tag{2.2}$$

where T_{R_H} is the temperature at the Hubble horizon, identified with the CMB temperature after the decoupling: $T_{R_H} = T_{CMB}$. This is a model-specific thermodynamic identification and is assumed here to be a general equilibrium identity.

2.3.b Wojnow's Cosmic Entropy Formula

Wojnow [10, 8] introduced the following expression for the thermodynamic cosmic entropy:

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$$S_{R_H} = 16\pi^2 k_B \left(\frac{T_{CMB}}{T_{Pl}}\right)\left(\frac{t_{R_H^2}}{t_{Pl}^2}\right) \tag{2.3}$$

where the Planck constants are defined as:

- Planck temperature: $T_{Pl} = \sqrt{\frac{\hbar c^5}{G\, k_B^2}}$
- Planck time: $t_{Pl} = \sqrt{\frac{\hbar\, G}{c^5}}$

2.3.c The Tatum, Seshavatharam, and Lakshminarayana Formula

$$T_{CMB} = T_{R_H} = \left(\frac{T_{Pl}}{8\pi}\right)\left(\sqrt{\frac{2 l_{Pl}}{R_H}}\right) \tag{2.4}$$

We use the H, T_{CMB} relationship Eq.2.4, proposed by Tatum, Seshavatharam, and Lakshminarayana, S. (2015), see [4]. This relationship has been demonstrated by Haug and Wojnow (2023), see [5], then published by Haug (2024), see [6].

2.4 Volume of the Hubble Sphere

The volume associated with the Hubble radius is:

$$V = \left(\frac{4}{3}\right)\pi\, R_H^3 \tag{2.5}$$

2.5 Critical Energy Density

In the $R_H = ct$ model, the critical energy density is given by:

$$\rho_{E,c} = \frac{3\, c^4}{8\pi\, G\, R_H^2} \tag{2.6}$$

This expression can also be written in terms of the CMB temperature [5,6], [7-8]:

$$\rho_{E,c} = 384 \frac{\pi^3 k_B^4 T_{CMB}^4}{\hbar^3 c^3} \tag{2.7}$$

3. Derivation of Pressure from Wojnow's Entropy

3.1 Thermodynamic Definition of Pressure

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Because both the entropy and the energy of the cosmological black-hole interior evolve with R_H, the equilibrium definition $P_{eff} = -(\partial E/\partial V)_S$ is not directly applicable along the cosmological trajectory.

We therefore define a model-specific effective pressure associated with the change of the enclosed black-hole volume by

$$\boldsymbol{P_{eff} = -\left(\frac{dE_{R_H}}{dV_H}\right)} \tag{3.1}$$

In the remainder of this document, (P_{eff}) denotes only this trajectory-defined effective quantity. It should not be identified, without an additional dynamical argument, with the equilibrium pressure $P = -\left(\frac{\partial E}{\partial V}\right)_S$.

This quantity represents the negative rate of change of the enclosed energy with respect to the geometrical interior volume along the $R_H = ct$ trajectory. It should not be confused with the standard thermodynamic pressure obtained from a partial derivative at constant entropy, nor with the pressure associated with a variable cosmological constant in extended black-hole thermodynamics.

3.2 Calculating Partial Derivatives

From equation (2.1), the energy derivative with respect to R_H is:

$$\boldsymbol{\frac{dE_{R_H}}{dR_H} = \frac{c^4}{2G}} \tag{3.2}$$

- **Source**: Direct derivative of eq. (1) $E_{R_H} = c^4 R_H / (2G)$
- **Origin**: This expression of E_{R_H} comes from the black hole-universe analogy in general relativity, well-established in the literature (Bekenstein, Hawking, and subsequent work on horizon thermodynamics)
- **Reference**: Melia (2021) *Thermodynamics of the $R_h = ct$ Universe: a simplification of cosmic entropy* [9] explicitly uses this analogy

From equation (2.5), the volume derivative with respect to R_H is:

$$\boldsymbol{\frac{dV_H}{dR_H} = 4\pi R_H^2} \tag{3.3}$$

These are derivatives along the linear cosmological trajectory parametrized by RH=ct.

3.3 Expression for Pressure in Terms of R_H

Using the chain rule for partial derivatives:

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$$P_{eff} = -\frac{\left(\frac{dE_{R_H}}{dR_H}\right)}{\left(\frac{dV}{dR_H}\right)} \tag{3.4}$$

These are derivatives along the linear cosmological trajectory parametrized by RH=ct.

$$P_{eff} = -\frac{\left(\frac{c^4}{2G}\right)}{4\pi\, R_H^2} \tag{3.5}$$

$$P_{eff} = -\frac{c^4}{8\pi\, G\, R_H^2} \tag{3.6}$$

3.4 Relation Between R_H, T_{CMB}, and S_{R_H}

From equations (2.1) and (2.2), we have $E_{R_H} = T_{CMB}S_{R_H} = \frac{c^4 R_H}{2G}$. Solving for R_H:

$$R_H = 2\, G\frac{T_{CMB}S_{R_H}}{c^4} \tag{3.7}$$

Squaring both sides:

$$R_H^2 = 4\frac{G^2T_{CMB}^2S_{R_H}^2}{c^8} \tag{3.8}$$

3.5 Expression of the Trajectory Effective Pressure in Terms of the Product $T_{R_H}\, S_{R_H}$

Substituting equation (3.8) into equation (3.6):

$$P_{eff} = -\frac{c^4}{8\pi\, G\left(4\frac{G^2T_{CMB}^2S_{R_H}^2}{c^8}\right)} \tag{3.9}$$

$$P_{eff} = -\, c^4 \cdot \frac{c^8}{32\pi\, G^3T_{CMB}^2S_{R_H}^2} \tag{3.10}$$

$$P_{eff} = -\frac{c^{12}}{32\pi\, G^3(T_{CMB}S_{RH})^2} \tag{3.11}$$

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This is the main algebraic result of the present note. The trajectory effective pressure can be expressed as a function of the product $T_{R_H}S_{R_H}$. Since $T_{R_H}S_{R_H} = E_{R_H}$, this relation is a re-expression of the geometrical result $P_{eff} = -c^4/(8\backslash\pi\ G\ R_H^2)$, rather than an independent derivation of pressure from entropy alone.

3.6 Alternative Expression: Pressure in Terms of Critical Energy Density

From equations (2.6) and (3.6) or from equation (3.11) with $T_{CMB}^2 S_{R_H}^2$ formulas (see [5] [10, 8]), we observe that:

$$\boldsymbol{P_{eff}} = -\frac{\boldsymbol{c^4}}{\boldsymbol{8\pi\ G\ R_H^2}} = -\left(\frac{\boldsymbol{1}}{\boldsymbol{3}}\right)\left(\boldsymbol{3}\frac{\boldsymbol{c^4}}{\boldsymbol{8\pi\ G\ R_H^2}}\right) \tag{3.12}$$

$$\boldsymbol{P_{eff}} = -\frac{\boldsymbol{\rho_{E,c}}}{\boldsymbol{3}} \tag{3.13}$$

This provides a relationship between pressure and critical energy density in this $R_H = ct$ thermodynamical model.

By the model-specific definition adopted in equation (3.1), the effective pressure satisfies

$$\boldsymbol{dE_{R_H}} = -\boldsymbol{P_{eff}\ dV} \tag{3.14}$$

Equation (3.14) is therefore an identity along the cosmological trajectory and is not presented as the complete first law of black-hole thermodynamics. In particular, it does not replace the usual horizon-entropy term involving the Hawking temperature T_{Hw} and the Bekenstein-Hawking entropy S_{BH}.

Since (P_{eff}<0) and (dV>0) during the expansion, this implies $dE_{R_H} > 0$,

$$\boldsymbol{E_{R_H}} = \frac{\boldsymbol{c^4 R_H}}{\boldsymbol{2G}} \tag{3.15}$$

consistent with the increase of the enclosed Hubble-scale energy in a black hole.

4. Interpretation of the Effective Negative Pressure

4.1 Vacuum Pressure in Standard ΛCDM Cosmology

In the standard ΛCDM cosmological model, dark energy is described by a cosmological constant Λ, which corresponds to a vacuum energy density ρ_{vac} .The equation of state for vacuum energy is characterized by a pressure $P_{vac}^{\Lambda CDM} \neq P_{eff}^{R_H=ct}$ that satisfies:

$$\boldsymbol{P_{vac}^{\Lambda CDM}} = -\boldsymbol{\rho_{vac}} \tag{4.1}$$

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This relationship arises from the thermodynamic properties of the vacuum, where the pressure is equal in magnitude but opposite in sign to the energy density. This negative pressure is responsible for the accelerated expansion of the universe.

In the ΛCDM framework:

- The vacuum energy density is constant: $\rho_{vac} = \Lambda \frac{c^4}{8\pi G}$
- The pressure is $P_{vac}^{\Lambda CDM} = -\rho_{vac}$ = *-Λ c4 / (8π G)*
- This corresponds to an equation of state parameter *w = P/ρ = -1*

4.2 Effective Pressure in Melia's $R_H = ct$ Cosmology

In Melia's $R_H = ct$ cosmological model, the pressure-dense relationship **was first rigorously derived by Melia [9]** using the Friedmann equations combined with the fundamental constraint $R_H = ct$ (H = 1/t). The key result, **P = -ρ/3**, emerges directly from cosmic dynamics and is not an ad hoc assumption.

Derivation by Melia [9]:
From the Friedmann equations for a flat FLRW universe (Melia don't use $\mathbf{P_{eff}}$):

$$\rho + P_{eff} = H^2 c^2/(4\pi G) = (2/3)\,\rho \Rightarrow P_{eff} = -\rho/3$$

This expression reveals **fundamental differences** with the standard ΛCDM model, with significant cosmological consequences:

1. **Different Equation of State:**
   The $R_H = ct$ model has *P = -ρ/3* (**w = -1/3**), whereas ΛCDM has *P = -ρ* (**w = -1**).
   Note: The relation P=- ρ/3 is therefore not claimed here as a new result. It is already a defining dynamical property of the $R_H = ct$ *Melia's framework. The contribution of the present work is instead to express the corresponding effective pressure explicitly in terms of Wojnow's thermodynamic entropy and the CMB temperature :* $\boldsymbol{P_{eff}} = -\frac{c^{12}}{32\pi\, G^3 T_{CMB}^2 S_{R_H}^2}$, *our Equation (3.11) repeated.*
2. **Physical Implications:**
   - In ΛCDM, dark energy with *w = -1* leads to exponential expansion (de Sitter phase), requiring a cosmological constant Λ.
   - In $R_H = ct$, the *w = -1/3* value corresponds to a **coasting universe** with linear expansion ($R_H \propto t$), a fundamental feature of Melia's model.
   - **The $\boldsymbol{R_H = ct}$ model does not require cosmic acceleration explain assacioted to a dark energy**, as the linear expansion naturally fits observational data without invoking Λ.
3. **Thermodynamic Consistency:**
   The relation [9] derived *P = -ρ/3* from cosmic dynamics, **under the model-specific thermodynamic and cosmological assumptions adopted here** (E = TS, P = -dE/dV) when applied to the Hubble horizon with Wojnow's entropy S_{R_H}. The present calculation provides a geometrical representation of the same relation through the model-specific definition.

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4. **Cosmological Consequences:**
   The equivalence $\rho_{E,c} = \rho_c\, c^2$ (Melia's critical density * c^2) confirms that the pressure expression *P = -ρ/3* in the $R_H = ct$ framework represents a fundamental property of the gravitational field at the Hubble horizon, maintaining the $R_H = ct$ constraint across all cosmic times **without additional dark energy components**.

4.3 Physical Interpretation

1. The relation $P_{eff} = -\rho_{E,c}/3$ follows from the trajectory definition $P_{eff} = -dE_{R_H}/dV_H$), together with $E_{R_H} \alpha\ R_H$ and $V_H\ \alpha\ R_H^3$. Wojnow's entropy does not independently generate this pressure. Rather, the identity $E_{R_H} = T_{R_H} S_{R_H}$ allows the same trajectory effective pressure to be parametrized by the thermodynamic product $T_{R_H} S_{R_H}$.
2. **Geometric Interpretation:** The factor of *1/3* arises from the spherical geometry of the Hubble horizon ($V \propto R_H^3$).

4.4 Comparison with Bekenstein-Hawking Entropy

It is instructive to compare Wojnow's entropy with the Bekenstein-Hawking entropy for a black hole:

- Bekenstein-Hawking: $S_{BH} = k_B c^3 A\,/\,(4\,\hbar\,G) = \pi\, k_B c^3 R_S^2\,/\,(\hbar\, G)$, where *A* is the horizon area ($A = 4\pi R_H^2$)
- Wojnow's entropy: $S_{R_H} = 16\pi^2 k_B^2 T_{CMB} R_H^2$ *c^(1/2) / (ħ^(3/2) G^(1/2))*

Both entropies are proportional to the horizon area (R^2). Wojnow's entropy incorporates the CMB temperature and Planck-scale physics, making it specifically adapted to cosmological applications within the Melia's $R_H = ct$ framework.

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5. Formulas Used in This Derivation

The relation $P_{eff} = -\rho_{E,c}\,/\,3$ is not claimed here as a new equation of state. The contribution of the present note is limited to showing that, under the adopted identity $E_{R_H} = T_{R_H} S_{R_H}$, the trajectory effective pressure may be written as

$\boldsymbol{P = -c^{12}\,/\,(32\pi\, G^3\, T_{CMB}^2\, S_{R_H}^2)}$ **(3.11)**
or equivalently:
$\boldsymbol{P = -\rho_{E,c}\,/\,3}$ **(3.13)**

This formula contains no information independent of the product $S_{R_H} T_{R_H} = E_{R_H}$ It provides a thermodynamic parametrization of the trajectory effective pressure.

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Equation Number	Formula	Description
(2.1)	$E_{R_H} = c^4 R_H / (2G)$	Energy at Hubble horizon (GR)
(2.2)	$E_{R_H} = T_{CMB} S_{R_H}$	Thermodynamic relation
(2.3)	$S_{R_H} = 16\pi^2 k_B \left(\frac{T_{CMB}}{T_{Pl}}\right)\left(\frac{t_{Rh}^2}{t_{Pl}^2}\right)$	Wojnow's entropy formula
(2.5)	$V = (4/3)\pi\, R_H{}^3$	Hubble sphere volume
(2.6)	$\rho_{E,c} = 3\, c^4 / (8\pi\, G\, R_H^2)$	Critical energy density
(2.7)	$\rho_{E,c} = 384\, \pi^3\, k_B^4\, T_{CMB}^4 / (\hbar^3 c^3)$	Critical density vs T_{CMB}
	$P = -\, (\partial E/\partial V)$	Pressure definition
(3.6)	$P = -\, c^4 / (8\pi\, G\, R_H^2)$	Pressure vs R_H
(3.11)	$\mathbf{P = -\, c^{12} / (32\pi\, G^3 \left(T_{CMB} S_{R_H}\right)^2)}$	**Main result: Pressure vs S_{R_H} and T_{CMB}**
(3.13)	$\mathbf{P_{eff} = -\, \rho_{E,c} / 3}$	**Pressure in $R_H = ct$ vs critical energy density**
(4.1)	$P_{vac} = -\rho_{vac}$	Vacuum pressure in ΛCDM.

6. Conclusion

The relation $P_{eff} = -\rho_{E,c}/3$ is not claimed here as a new equation of state. Under the adopted identity $E_{R_H} = T_{CMB} S_{R_H}$, the trajectory effective pressure may be expressed as a function of the product $T_{CMB} S_{R_H}$. This relation is a re-expression of the geometrical trajectory result and does not constitute an independent derivation of pressure from entropy alone.**P = - c¹² / (32π G³ $\left(T_{CMB} S_{R_H}\right)^2$)**together with the entropy scaling

$S_{R_H}^{2/3} \propto R_H \propto t.$

Or equivalently:

$$\boldsymbol{P_{eff}^{(Melia)} = P_{eff}^{(traj)} = -\, \rho_{E,c} / 3}$$

This pressure with **distinct interpretations** in the standard ΛCDM model (where $P = -\rho$) but compatible with Melia's $R_H = ct$ model (where $P = -\rho/3$ *and* $\rho^{(Melia)} = \rho_{E,c}$) . In the Wojnow representation, this effective pressure accompanies the expansion of the Hubble volume V_H while T_{R_H} decreases and the enclosed energy E_{R_H}, increases according to the adopted $R_H = ct$ thermodynamic relations. This is the consequence, with our hypothesis, of a coupled thermodynamic evolution of T_{R_H} and S_{R_H}in a growing volume and a growing energy

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system. This coupled evolution is consistent with equation (3.11), $dE_{R_H} = -P_{\text{eff}}\, dV$, so that the increase of the enclosed energy accompanies the expansion of the Hubble volume under the adopted effective-pressure description.

Under the model-specific thermodynamic and cosmological assumptions adopted here, the resulting relations are internally consistent. The factor of *1/3* in the $R_H = ct$ model distinguishes it from the standard ΛCDM framework and is a direct consequence of the geometric and thermodynamic properties of the Hubble horizon in this alternative cosmological paradigm.

The scaling structure identified here may motivate the search for analogous thermodynamic systems in which a decreasing temperature and an increasing entropy combined to produce an increasing internal energy. Whether such analogues exist outside cosmology remains an open question.

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