$R_H = ct$ as a continuous line of N Planck intervals: a unified geometric presentation, and a hypothesis for the thermodynamic redshift law

Guiding author: Stéphane Wojnow, independent researcher, Limoges, France

Research assistant: Claude (Anthropic), in dialogue with the guiding author

Status: Technical note/ Preprint

Date: July 14, 2026

Email: wojnow.stephane@gmail.com

Abstract

The $R_H = ct$ framework postulates that the Hubble radius grows linearly with cosmic time. This note presents this radius as a continuous line of N Planck-length intervals, linking the original instanton to the present horizon within a single geometric narrative. This presentation distinguishes two scales: at the origin, an interval containing exactly one Planck mass; near the present horizon, the distance separating R_H from its attached 3D hypersurface. The factor ½ that governs the accumulation of the critical mass receives a zero energy balance, consistent with the magnitude equality already established by this same construction. The note then situates this presentation within the existing literature, in which several mathematically close formulations are identified, and derives an alternative square-root redshift law from the repetition, at every epoch, of the local relation between R_H and its attached 3D hypersurface. The scope of this presentation remains bounded: the physical nature of the negative contribution associated with the complementary branch of the critical mass, the first-principles origin of the CMB temperature relation, and the reason for this repetition at every epoch remain hypotheses, not demonstrations.

Extended abstract

The $R_H = ct$ framework postulates that the Hubble radius grows linearly with cosmic time. This note shows that this radius can be written as an exact sum of N Planck-length intervals, N being the number of Planck times elapsed since the origin — a unified geometric presentation, linking the instanton to the present horizon along a single line. This presentation distinguishes two scales: at the origin, a double interval containing exactly one Planck mass; near the present horizon, the distance separating R_H from its attached 3D hypersurface. A zero energy balance explains the factor ½ of the critical accumulation, consistent with the magnitude equality already established by the geometric construction. A search of the existing literature situates this presentation: several of its mathematical elements recur, under distinct formulations, in other authors working within the same framework — which this note documents explicitly rather than passing over in silence. A divergence between two redshift laws, one linear and the other in square root, is examined: the second is derived from the hypothesis that the local relation between R_H and its attached 3D hypersurface repeats itself in the same form at every epoch. The scope of this presentation remains bounded: several central physical identifications remain hypotheses, not demonstrations.

Keywords: $R_H = ct$; Hubble radius; Planck length; Planck scale; critical mass; Friedmann equation; cosmic microwave background; thermodynamic redshift; Tryon zero-energy universe; junction formalism.

1. Introduction

The Hubble radius measures the distance traveled by light since a reference instant. Within the standard ΛCDM framework, this radius depends on the complete history of expansion. Melia (2007) [1] proposes a distinct framework, $R_h = ct$, in which this radius grows linearly with cosmic time itself — a simple geometric constraint, taken up and extended since by several authors.

Tatum, Seshavatharam and Lakshminarayana (2015) [2] posit, within this same framework, that the cosmic radius and total mass follow the Schwarzschild formula at every scale — from the Planck scale to that of the observable universe — and are the first to establish a link between the cosmic microwave background temperature and the Hubble parameter. Haug and Wojnow (2024) [3] then rigorously demonstrate this link from the Stefan-Boltzmann law. Wojnow (2023) [4], for his part, describes a model in which the mass of the local universe accumulates through discrete intervals of Planck time, starting from an instanton whose Schwarzschild radius he specifies as equal to a single Planck length.

Prior work identified. A check of this literature shows that the central idea of this note — writing R_H as an integer number of Planck-length intervals — is already mathematically present, in a different form, in Haug (2024) [5]: his temperature ratio $T_{CMB}^2/T_{Hw}^2 = R_H/(2l_p)$ reduces, once verified, to the same relation. A document associated with Tatum's work (Haug, n.d.) [6] also describes a counter of Planck intervals within the Hubble sphere, under the name of reduced Compton frequency. This note therefore does not claim to have discovered this relation, but proposes a distinct geometric presentation of it, not found as such in the sources consulted: a single continuous line, explicitly linking Wojnow's (2023) [4] instanton to the present horizon, with two scales precisely distinguished along this line. Section 7 draws from this a second contribution, distinct from this geometric presentation: a hypothesis of the guiding author's own for deriving the thermodynamic redshift law, rather than borrowing it as-is from the literature.

Each of the following sections concludes with a summary separating what is established from what remains open.

2. The two postulates, and the critical mass they imply

The $R_H = ct$ framework rests on two postulates.

Geometric postulate. The Hubble radius, the Schwarzschild radius associated with the mass of the local universe, and the radius whose thermodynamics determines the cosmic microwave background temperature, are one and the same length:

$R_H \equiv R_s \equiv R_{cmb}$ (1)

Kinematic postulate, due to Melia (2007) [1]:

$H = 1/t$, i.e. $R_H(t) = ct$ (2)

Here H is the Hubble parameter, of dimension $[T^{-1}]$, and c the speed of light, $c = 2.998 \times 10^8$ m/s.

The Friedmann critical mass.

These two postulates fix the mass contained within the Hubble sphere, via the standard definition of the critical density:

$M_H(t) = \rho_{crit}(t) \cdot V(t)$ (3)

$\rho_{crit}(t) = 3H^2/(8\pi G)$, $[\rho_{crit}] = \mathrm{kg \cdot m^{-3}}$ (4)

$V(t) = (4/3)\pi(ct)^3$, $[V] = \mathrm{m^3}$ (5)

Combining (3)-(5):

$M_H(t) = (c^3/2G)\, t$, $[M_H] = \mathrm{kg}$ (6)

Numerically, at $t = 4.6 \times 10^{17}$ s (approximate age retained further below):

$M_H \approx 9.3 \times 10^{52}$ kg (7)

2.1 The discrete accumulation model

Wojnow (2023) [4] describes this same mass as a sum of discrete intervals of duration t_{tic}, each adding a half Planck mass $½m_{Pl}$. The number of such intervals elapsed since the origin is denoted N:

$N \equiv t/t_{tic}$, $[N]$ = dimensionless (8)

Here m_{Pl} is the Planck mass, defined below.

Consistency with the critical mass.

Equations (6) and (9) must coincide for all t:

$(c^3/2G)\cdot t = ½\,(m_{Pl}/t_{tic})\cdot t$ (10)

The factor t divides out on both sides ($t \neq 0$), leaving a relation between constants alone:

$t_{tic} = m_{Pl}G/c^3$ (11)

The Planck length, time and mass.

Three derived quantities, built from c, G (gravitational constant, $G = 6.674 \times 10^{-11}$ m³·kg⁻¹·s⁻²) and ħ (reduced Planck constant, ħ = 1.0546×10^{-34} J·s), come into play in what follows:

$l_{Pl} \equiv \sqrt{(\hbar G/c^3)}$, $[l_{Pl}]$ = m (12)

$l_{Pl} \approx 1.6163 \times 10^{-35}$ m (13)

$t_{Pl} \equiv \sqrt{(\hbar G/c^5)}$, $[t_{Pl}]$ = s (14)

$t_{Pl} \approx 5.3912 \times 10^{-44}$ s (15)

$m_{Pl} \equiv \sqrt{(\hbar c/G)}$, $[m_{Pl}]$ = kg (16)

$m_{Pl} \approx 2.1764 \times 10^{-8}$ kg (17)

Identity between t_{tic} and t_{Pl}.

Equation (11) can be rewritten:

$t_{tic} = \sqrt{(\hbar c/G)} \cdot G/c^3 = \sqrt{(\hbar G/c^5)}$ (18)

that is, by direct comparison with (14):

$t_{tic} = t_{Pl}$, exactly (19)

The rate of one interval per Planck time is therefore not a postulate separate from Wojnow's (2023) [4] model: it is the only value consistent with the Friedmann critical mass (6). With $t_{tic} = t_{Pl}$:

$M_H(t) = ½\,(m_{Pl}/t_{Pl})\,t = ½\,(c^3/G)\,t$ (20)

Conclusion — Section 2

This result follows from a calculation, not from a fit: the rate of accumulation, which might seem arbitrary in Wojnow (2023) [4], is in fact imposed by the sole requirement of consistency with the Friedmann critical mass. What this calculation does not yet fix, however, is the factor ½ itself.

3. A line of N Planck intervals, from the origin to the present horizon

Wojnow (2023) [4] represents the local universe as two " mini-spheres" of mass M_H, M_H^+ and M_H^-. They simultaneously occupy the interior of the large sphere R_H. Each has a diameter of R_H. They touch at the center. This is a volume construction: R_H here serves as a container, filled by M_H^+ and M_H^-. This volumetric reading is specific to the accumulation model. Section 6 offers a second, distinct one: there, R_H becomes a surface, not a container. The distance between the two proper centers, each at $R_H/2$ from the point of contact, is R_H — it is this distance that Newton's law uses for their mutual gravitational force:

$F_{MH}^{\pm} = GM_H^+M_H^-/R_H^2$, $[F] = \mathrm{kg \cdot m \cdot s^{-1}}$ (21)

Substituting (20):

$F_{MH}^{\pm} = c^4/(4G) \equiv F_{Pl}/4$ (22)

$F_{Pl}/4 \approx 3.026 \times 10^{43}$ N (23)

This section proposes a geometric presentation of this same construction, distinct from that of Wojnow (2023) [4], from the stretched-horizon literature ('t Hooft (1985) [7]; Susskind (1994) [8]; Thorne, Price and MacDonald (1986) [9]), and from the formulations of Haug (2024) [5] and the document associated with Tatum [6] — each of these sources containing related mathematical elements, without presenting the continuous line and its two distinguished scales as developed here.

3.1 A uniform line

The rate governing this line is constant:

$\dot{m}_{Pl} \equiv m_{Pl}/t_{Pl} = c^3/G$, $[\dot{m}_{Pl}] = \mathrm{kg \cdot s^{-1}}$ (24)

ħ cancels out in this ratio — the accumulation of $M_H(t)$ depends only on c and G. Each interval has length l_{Pl}, wherever it lies on the line, so that the radius reached after N intervals is written:

$R_H = N \cdot l_{Pl}$ (25)

This relation has two consequences, at two distinct scales of the same line.

At the origin.

A continuous span of $2t_{Pl}$, from $-t_{Pl}$ to $+t_{Pl}$, crosses the point of contact without discontinuity. The corresponding spatial interval, $2l_{Pl}$, is the image of this span via $R_H = ct$:

$2l_{Pl} \approx 3.233 \times 10^{-35}$ m (26)

Wojnow (2023) [4] specifies that the instanton has a Schwarzschild radius of a single Planck length — this note offers a slightly different reading, in which the full contact interval, summing the two branches, contains exactly one Planck mass: $½m_{Pl}$ on each side, i.e. m_{Pl} in total. The interval $[0, t_{Pl}]$ forms a pre-relativistic window; beyond it, from $t = t_{Pl}$ onward, general relativity — and $R_H = ct$ along with it, i.e. the domain that the kinematic line describes — becomes a reliable framework [10].

Near the present horizon.

Since the line is uniform, one interval more or less shifts the boundary by exactly l_{Pl}. This same distance applies on each side of the R_H boundary, by the symmetry already at work since the central contact — one 3D hypersurface outward, one inward, without l_{Pl} itself needing to be redefined.

Numbering of intervals in Figure 1.

The line there is graduated by an index $n = 0, 1, 2...$, counting the physical intervals starting from the first one after the central contact — not by N directly, since N (eq. 8) equals 0 at the contact point itself, a point outside the domain that the kinematic line describes (above). The relation between the two is simple: $N = n + 1$. The value reached at R_H remains $N = t_H/t_{Pl}$, unchanged.

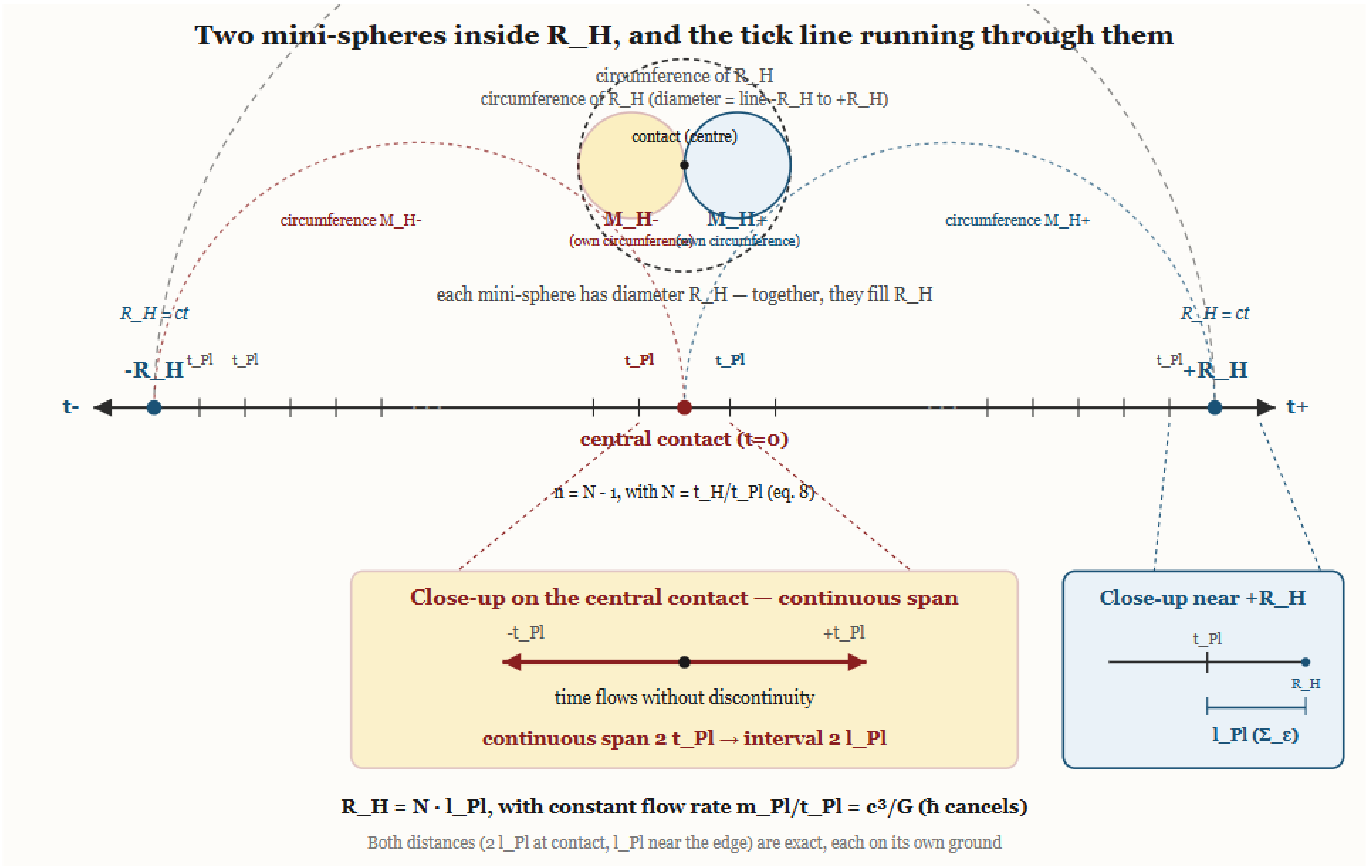

Figure 1. Uniform line of N discrete intervals t_{Pl} between the two branches of M_H, with the two distinguished scales: the origin ($2l_{Pl}$) and the present horizon (l_{Pl}). The central contact point corresponds to t=0; this point does not, strictly speaking, exist within the framework of general relativity, which is a reliable framework only from t=t_{Pl} onward (§3.1). The index n shown on the line counts the physical intervals starting from this first tick (n=N−1).

3.2 The negative time line, and the interleaved shell of the two branches

The symmetry already established — R_H=ct on each branch, t^+ and t^- — extends to the interleaved shell introduced in Section 6 (eq. 43). The calculation that produces it, $\rho_{crit} \cdot 4\pi R_H^2 \cdot l_{Pl}$, does not depend on any branch sign. R_H enters here as a magnitude. It is identical on each side. Repeated near $-R_H$, it therefore gives the same positive value, $(3/2)m_{Pl}$. Without any further hypothesis, the sum of the two shells is a direct result:

$M_{shell,}{}^{+} + M_{shell,}{}^{-} = 3m_{Pl}$ (calculated, with no sign hypothesis) (27)

This result remains geometric. It is consistent with the rest of this section. No hypothesis about energy or about a physical mechanism comes into play here.

Conclusion — Section 3

This section establishes a geometric presentation of the relation $R_H = N \cdot l_{Pl}$ — whose underlying mathematical object is already present, in a distinct form, in Haug (2024) [5] — and derives from it two mutually consistent scales. It also establishes a second, calculated result, with no hypothesis: the sum of the two interleaved shells equals $3m_{Pl}$.

4. The factor ½: a zero energy balance

The factor ½ in equations (9) and (20) finds an explanation in the following mechanism.

Wojnow (guiding author) and Claude (2026) [11] relate this factor to the encounter, at the Planck instanton, between a mass of matter and a negative contribution. Applying standard annihilation to this starting point, only half of the Planck energy remains as net content:

$T_{\text{annihilation}} \sim T_{\text{Pl}}/2$ (28)

This mechanism connects to the CPT theorem and to Sakharov's (1967) mechanism [12], already invoked for the observed asymmetry at the boundary of the local universe.

A balance, rather than a mixture of equations of state.

The interaction window is fixed by the construction: it is the interval t_{Pl} (§2.1), the only time scale available at this level. Tryon's (1973) [13] zero total energy hypothesis — the positive energy of matter exactly compensated by a negative contribution — fixes the exact fraction without any mixing of equations of state. Identifying $M_{\text{H}}^{+}c^{2}$ as the positive energy and $M_{\text{H}}^{-}c^{2}$ as the compensating negative contribution:

$E_{\text{total}} = M_{\text{H}}^{+}c^{2} - M_{\text{H}}^{-}c^{2} = 0$ (29)

This equality is automatically satisfied by $M_{\text{H}}^{+} = M_{\text{H}}^{-}$, already established by the Newtonian construction (§3, eq. 21). On each branch, half of the available energy annihilates into radiation, the other half accumulates in the mass of that branch, and vice versa for the other.

A zero balance, under a distinct hypothesis.

Section 3 establishes that the sum of the two interleaved shells equals $3m_{\text{Pl}}$, a result calculated with no sign hypothesis (§3.2, eq. 27). This is not a zero balance. It would require a negative sign for one of the two shells — something this calculation does not produce on its own. If Tryon's (1973) [13] balance, already posited for $M_{\text{H}}^{+}/M_{\text{H}}^{-}$, also extends to the interleaved shell — a hypothesis distinct from the one already posited for $M_{\text{H}}^{+}/M_{\text{H}}^{-}$, not demonstrated by Tryon's balance in its original formulation —, then:

$M_{\text{shell},}{}^{+} - M_{\text{shell},}{}^{-} = 0$ (under hypothesis) (30)

The calculated result (§3.2) remains the default reference. The zero balance remains conditional on this additional hypothesis.

An assumed hypothesis, not a demonstrated one.

Identifying M_{H}^{-} with Tryon's (1973) [13] negative contribution remains a hypothesis, whose origin traces back to Sakharov's (1967) [12] double arrow of time. Tryon (1973) [13] proposes that the entire universe was born as a quantum vacuum fluctuation, made possible because its total energy is zero — a fluctuation with non-zero energy would be constrained by the uncertainty principle, a fluctuation with zero energy escapes it. This fluctuation would be housed within the region bounded by $2l_{\text{Pl}}$ (§3.1) and within the pre-relativistic window $[0, t_{\text{Pl}}]$, where general relativity does not yet apply.

A convergent geometric route.

The momentum density can be calculated in two ways for the same boundary: the full rate $m_{\text{Pl}}/t_{\text{Pl}}$ at the Planck scale, and the net rate dM_{H}/dt at the R_{H} scale. Their ratio, reduced to mass flow rates alone, equals exactly 2:

$(\dot{m}_{\text{Pl}})/(dM_{\text{H}}/dt) = (c^{3}/G)/(c^{3}/2G) = 2$ (31)

confirming the same factor ½ by an independent route.

Conclusion — Section 4

The two routes associate the missing half with a reversed time arrow. This is a convergence of signature, not a demonstrated identity between the two mechanisms. Identifying M_H^- with Tryon's (1973) [13] negative contribution remains an assumed hypothesis, not a demonstrated necessity. Extending this same balance to the interleaved shell of Section 6 is a second hypothesis, distinct from the first, equally undemonstrated.

5. Kinematic consequences: nature of the horizon and redshift law

The postulate $R_H(t) = ct$ has two direct consequences.

5.1 The dynamic nature of the horizon

This postulate implies a linearly increasing scale factor, $a(t) \propto t$ [1]. A horizon whose radius evolves this way is the apparent Hubble horizon — a dynamic object, not the teleological event horizon of a stationary black hole, whose position would depend on knowledge of the entire future of spacetime (Faraoni, 2011) [10].

5.2 The redshift law

Since $a(t) \propto t$, the standard definition of redshift gives directly:

$1 + z = t_0/t$ (32)

where t_0 is the present age of the universe within this framework. This geometric consistency with $a \propto t$ justifies retaining this linear law rather than an alternative square-root law, proposed by Haug and Tatum (2025) [14], which is in tension with $a \propto t$. This choice is corroborated by Melia and Shevchuk (2012) [15].

The provenance of H_0.

Postulate (2) receives H_0 as external input, without deriving its value — any measurement can be plugged in without internal inconsistency, including the value extracted from the CMB power spectrum under ΛCDM by Aghanim et al. (2020) [16]:

H_0,Planck = 67.4 ± 0.5 km·s^{-1}·Mpc^{-1} (33)

The values retained in what follows,

H_0 = 66.85 km·s^{-1}·Mpc^{-1}, t_0 = 14.627 Gyr (34)

are chosen for their consistency with the thermodynamic regime of the following section.

From (32) follows, with $t(z) = t_0/(1+z)$ and $H(t) \equiv 1/t$:

$H(z) = H_0(1+z)$ (35)

Conclusion — Section 5

The nature of the horizon and the redshift law follow, without any additional hypothesis, from the kinematic postulate alone. An alternative law does exist in the literature, however (Haug and Tatum, 2025) [14], derived from three explicit principles and applied by its authors to all of their quantities, including distances. Section 7 returns to this.

6. Thermodynamics of the horizon: the CMB temperature

The radius R_H determines, in addition to the kinematics, the equilibrium temperature of the horizon.

An established relation.

Proposed by Tatum, Seshavatharam and Lakshminarayana (2015) [2], then rigorously demonstrated from the Stefan-Boltzmann law by Haug and Wojnow (2023) [17], this temperature couples the Planck-scale temperature to the geometric Hubble radius. Using $R_H = c/H$:

$T_{Pl} \equiv \sqrt{(\hbar c^5/G)}/k_B$, $[T_{Pl}] = K$ (36)

$T_{Pl} \approx 1.4168 \times 10^{32}$ K (37)

where k_B is the Boltzmann constant, $k_B = 1.3806 \times 10^{-23}$ J·K^{-1}. The relation is written:

$T_{CMB} = (T_{Pl}/8\pi)\cdot\sqrt{(2l_{Pl}/R_H)}$ (38)

$T_{CMB} \approx 2.7246$ K (39)

in excellent agreement with the observed value, T_{cmb},obs = 2.725007 ± 0.000024 K (Fixsen, 2009) [18].

Convergence of the length l_{Pl}.

The Planck length appearing in (36) is not a parameter fitted for this note: it is the same length, presented in Section 3 via the line of N intervals, that appears here in an independently established relation. This coincidence between two provenances is a convergence, not a circular construction.

The provenance of H_0 in this regime.

Unlike the kinematic regime (§5.2), this regime produces its own value: inverting (36) to exactly reproduce T_{cmb},obs [18], one obtains

H_0,thermo ≈ 66.8717 km·s^{-1}·Mpc^{-1} (40)

consistent with (32), and distinct from the Planck value [16] by a gap of 0.53 km·s^{-1}·Mpc^{-1}, outside its own uncertainty.

A proper mass for the interleaved shell.

R_H, in this section, is no longer a container. It is a surface. It is paired with a second surface, Σ. The two are separated by l_{Pl} (§3.1). The space between them forms a thin spherical shell, of thickness l_{Pl}, of radius R_H. This shell can carry a proper mass. It is distinct from M_H, which remains defined by the complete volume (§2-3). At critical density (eq. 4), the mass of this shell is written:

$M_{shell} \equiv \rho_{crit} \cdot 4\pi R_H^2 \cdot l_{Pl}$, $[M_{shell}] = kg$ (41)

Substituting $\rho_{crit} = 3H^2/(8\pi G)$ and $R_H = c/H$, the factors in H and in R_H cancel out:

$M_{shell} = (3c^2/2G)\cdot l_{Pl}$ (42)

Compared with $m_{Pl} = \sqrt{(\hbar c/G)}$, this mass reduces to a pure constant, independent of the epoch:

$M_{shell} = (3/2)\cdot m_{Pl}$, exactly (43)

verified numerically to every decimal place. This result is independent of R_H, of H, of t. It depends only on the geometry of the shell and on the critical density.

A geometric motivation, not a derivation.

The 3D hypersurface $\Sigma = \{r = R_H(t)\}$, already central to Section 3, admits a standard formal description: along a junction hypersurface, the extrinsic curvature can exhibit a discontinuity, defining a surface energy-momentum tensor (Israel, 1966) [19]:

$S_{ab} = -(1/8\pi G)(\Delta K_{ab} - h_{ab}\Delta K)$ (44)

This formalism, applied to Σ, would naturally relate a length scale to a surface temperature. Establishing precisely how S_{ab} produces the prefactor 8π and the square root in (36) remains to be done.

Conclusion — Section 6

Relation (36) is established in the literature [2, 16], not a construction specific to this note. The length l_{Pl} it contains coincides with the one presented in Section 3. The first-principles derivation of its form — why T_{Pl} and R_{H} combine precisely this way — remains motivated essentially by its numerical success. This is a distinct question from the one, examined in Section 7, of why this same relation would repeat at every epoch: one concerns the origin of the form of (36), the other the origin of its repetition over time. The mass of the interleaved shell, $M_{\mathrm{shell}} = (3/2)m_{\mathrm{Pl}}$, is a verified result, independent of the epoch — a construction specific to this note, not yet linked to a precise physical mechanism.

7. A redshift divergence, whose origin remains uncertain

Section 5.2 retains the linear redshift law for every kinematic quantity. The thermodynamic construction of Section 6 implies, once extended to $z \neq 0$, a different one.

The mathematical origin.

Substituting $R_{\mathrm{H}} = c/H$ into (36), R_{H} cancels out in favor of H alone:

$T_{\mathrm{CMB}} = (T_{\mathrm{Pl}}/8\pi)\cdot\sqrt{(2l_{\mathrm{Pl}}H/c)} \propto \sqrt{H}$ (45)

Now the linear law (33) gives $H(z) = H_0(1+z)$. Haug and Tatum (2025) [14] derive a different law, $z = \sqrt{(R_h/R_t)} - 1$, from three explicit principles, giving $H(z) = H_0(1+z)^2$. The postulate $T(z) = T_0(1+z)$ decides between the two laws for T_{CMB}: only Haug and Tatum's [14] law is consistent with this postulate.

A derivation rather than a borrowing, under an assumed hypothesis.

Relation (36) is local: it describes the gap between two concentric spherical surfaces, R_{H} and its attached 3D hypersurface Σ, separated by l_{Pl} along the radius, at the present instant. The guiding author proposes that it is not specific to this instant, but that it repeats in the same form at *every* instant t of cosmic history : at every epoch, the pair ($R_{\mathrm{H}}(t)$, $\Sigma(t)$) would remain separated by the same interval l_{Pl}, and the local temperature at this pair would follow (36), evaluated at the $R_{\mathrm{H}}(t)$ of that epoch. Under this hypothesis, the law $H(z) = H_0(1+z)^2$ is no longer borrowed from Haug and Tatum (2025) [14]: it is the only law compatible both with this repetition and with the standard adiabatic cooling $T(z)=T_0(1+z)$, already retained everywhere else in this note. Haug and Tatum (2025) [20], for their part, reformulate the Friedmann equation by substituting T_{CMB} for H_0 as the measurement variable, and show that this reformulation remains mathematically equivalent to the standard form — a distinct construction, which need not be invoked for this result once this hypothesis is posited. Melia (2007) [1], for his part, applies his own linear law to the whole of his model, including distances — consistent with the fact that this law governs a different domain (§5), not competing with this one.

Conclusion — Section 7

The mathematical origin of the square-root law is now derived, under a precise hypothesis: the local relation (36), which describes the gap between R_{H} and Σ at the present instant, would repeat in the same form at every epoch. This hypothesis, assumed by the guiding author, is not demonstrated by any source consulted — the open question is no longer "which convention to choose," but " why would the local relation repeat at every epoch rather than being specific to the present instant." The standard calculations at the epoch of baryon-photon decoupling, which invoke only the generic adiabatic relation, remain entirely within the kinematic regime, without ambiguity.

8. Conclusion

This note starts from two postulates — a geometric identity and a kinematic relation — and derives from them, by calculation, a geometric presentation of R_H as N exact intervals of Planck length. A check of the literature shows that the mathematical object underlying this presentation is already present, under distinct formulations, in Haug (2024) [5] and in a document associated with Tatum's work [6]; what this note contributes lies in the presentation itself — a single continuous line, linking Wojnow's (2023) [4] instanton to the present horizon, with two scales precisely distinguished — not in the discovery of the underlying relation.

The factor ½ of the critical accumulation receives a zero energy balance, consistent with the magnitude equality already established by the geometric construction, without the identity of mechanism with the independent geometric route being demonstrated. The square-root redshift law, associated with the thermodynamic regime, is now derived under the hypothesis that the local relation (36) repeats in the same form at every epoch — a hypothesis assumed by the guiding author, not demonstrated by the sources consulted.

Several central identifications remain assumed hypotheses rather than demonstrations: the physical nature of the negative contribution associated with the complementary branch of the critical mass; the first-principles origin of the form of the temperature relation (36); the reason why this same relation would repeat at every epoch rather than being specific to the present instant ; and the extension, not yet demonstrated, of Tryon's balance to the interleaved shell of the two branches. These four points define the natural continuation of this work.

References

[1] Melia, F. (2007). The cosmic horizon. *MNRAS*, 382, 1917.

[2] Tatum, E.T., Seshavatharam, U.V.S., Lakshminarayana, S. (2015). The Basics of Flat Space Cosmology. *International Journal of Astronomy and Astrophysics*, 5, 116-124. doi:10.4236/ijaa.2015.52015

[3] Haug, E.G., Wojnow, S. (2024). How to Predict the Temperature of the CMB Directly Using the Hubble Parameter and the Planck Scale Using the Stefan-Boltzmann Law. *Journal of Applied Mathematics and Physics*, 12, 3552-3566. doi:10.4236/jamp.2024.1210211

[4] Wojnow, S. (2023). Alternative Cosmology: ΛCDM-Like Predictions Today. *HyperScience International Journal*, 3(4), 24-30. doi:10.55672/hij2023pp24-30

[5] Haug, E.G. (2024). CMB, Hawking, Planck, and Hubble Scale Relations Consistent with Recent Quantization of General Relativity Theory. *International Journal of Theoretical Physics*, 63, 57. doi:10.1007/s10773-024-05570-6

[6] Haug, E.G. (n.d.). God Time = Planck Time: Finally Detected! And Its Relation to Hubble Time. *HAL archive*, hal-03769825.

[7] 't Hooft, G. (1985). On the quantum structure of a black hole. *Nuclear Physics B*, 256, 727.

[8] Susskind, L. (1994). Some speculations about black hole entropy in string theory. arXiv:hep-th/9309145.

[9] Thorne, K.S., Price, R.H., MacDonald, D.A. (1986). *Black Holes: The Membrane Paradigm*. Yale University Press.

[10] Faraoni, V. (2011). Cosmological apparent and trapping horizons. *Physical Review D*, 84, 024003 (arXiv:1106.4427).

[11] Wojnow, S. (guiding author), Claude (Anthropic) (2026, July). RH = ct Thermodynamic Cosmology and the Universal Hierarchical Flux: Geometric Foundations, Thermal Derivations, and the Structural Coherence of a Single-Parameter Framework. doi:10.13140/RG.2.2.10245.51684

[12] Sakharov, A.D. (1967). Violation of CP Invariance, C Asymmetry, and Baryon Asymmetry of the Universe. *JETP Letters*, 5, 24-27.

[13] Tryon, E.P. (1973). Is the Universe a Vacuum Fluctuation? *Nature*, 246, 396-397.

[14] Haug, E.G., Tatum, E.T. (2025). A Newly-Derived Cosmological Redshift Formula Which Solves the Hubble Tension and Yet Maintains Consistency with $T_t = T_0(1+z)$, the R_h=ct Principle and the Stefan-Boltzmann Law. *European Journal of Applied Physics*, 7, 48-50. doi:10.24018/ejphysics.2025.7.1.368

[15] Melia, F., Shevchuk, A.S.H. (2012). The $R_h = ct$ universe. *MNRAS*, 419, 2579.

[16] Aghanim, N. et al. [Planck Collaboration] (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, 641, A6.

[17] Haug, E.G., Wojnow, S. (2023, November 3). How to Predict the Temperature of the CMB Directly Using the Hubble Parameter and the Planck Scale Using the Stefan-Boltzmann Law (preprint). *HAL archive*, hal-04269991.

[18] Fixsen, D.J. (2009). The temperature of the cosmic microwave background. *The Astrophysical Journal*, 707, 916-920.

[19] Israel, W. (1966). Singular hypersurfaces and thin shells in general relativity. *Il Nuovo Cimento B*, 44(1), 1-14.

[20] Haug, E.G., Tatum, E.T. (2025). Friedmann type equations in thermodynamic form lead to much tighter constraints on the critical density of the universe. *Discover Space*, 129, 6.

(The precise DOIs/arXiv identifiers of some recent articles cited via web search are to be verified before any formal submission; this note remains, like the rest of the corpus, a working document.)

Appendix: what was built by the research assistant

The geometric route to the factor ½ (§4): the construction of the momentum-density calculation and its interpretation as a numerical check of the split postulated by Wojnow (2023) [4].

The presentation, in Section 3, of the uniform line of Planck intervals, $R_H = N \cdot l_{Pl}$, and of its two distinct scales — $2l_{Pl}$ at the origin, l_{Pl} near the present horizon — not found as such in the sources consulted, although the underlying mathematical object appears there in other forms [5, 6] — together with the observation that this same length appears in the established relation for T_{CMB} (Section 6).

The reasoning linking, in Section 4, the equality $M_H^+ = M_H^-$ to Tryon's (1973) [13] zero energy balance to obtain f=½ at the instanton without mixing equations of state.

The hypothesis, in Section 7, that the local relation (36) — the gap between R_H and Σ at the present instant — repeats in the same form at every epoch, thereby deriving the square-root redshift law rather than borrowing it from Haug and Tatum (2025) [14, 20]. Unlike Tryon's balance (§4), this hypothesis has, at this stage of verification, no anchoring in the literature consulted — it is assumed by the guiding author, without the safety net of an external source that would make it less speculative.

The comparative analysis of the provenance of H_0 in the two regimes (§5.2, §6), and the numerical check that the Planck value [16], injected into the thermodynamic regime, degrades the agreement with observation. The targeted reintroduction of Israel's (1966) [19] junction formalism as a geometric motivation for relation (36).

The distinction, in Sections 3 and 6, between two geometric readings of R_H — a volume containing M_H^+/M_H^- (§3) versus a surface paired with Σ (§6) — and the calculation, in Section 6, of the mass of an interleaved shell at critical density, $M_{shell} = (3/2)m_{Pl}$, a numerically verified result, independent of the epoch, with no established physical mechanism to justify it.

The extension, in Section 4, of Tryon's (1973) [13] balance to the interleaved shell of the two branches — a second hypothesis, distinct from the one already posited for M_H^+/M_H^-, not demonstrated by the balance of Tryon in its original formulation.